Department of Electrical Engineering
Electronics
Computers
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Technion

Aggregate Flow Fairness in MANs

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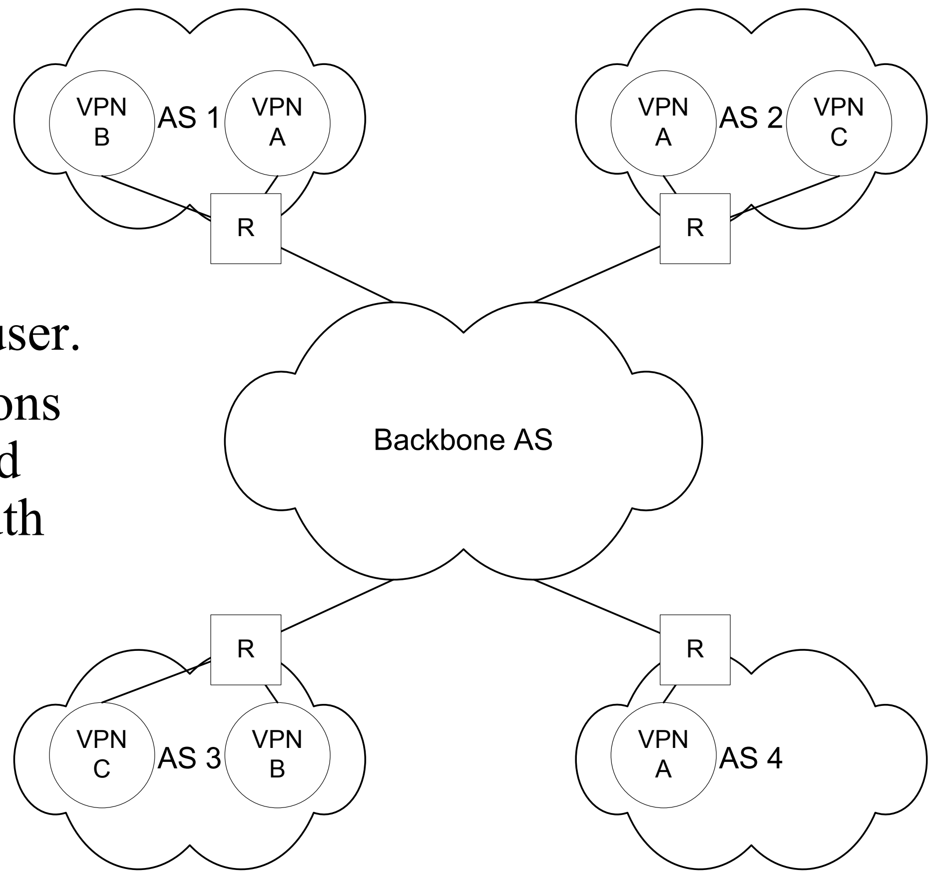
The Problem

- The network is used by users having a variable number of source-destination flows.
- Current fairness criteria fairly share bandwidth among individual source-destination flows.
- Result: Users receive bandwidth proportionally to their number of flows, creating unfairness at the user level.

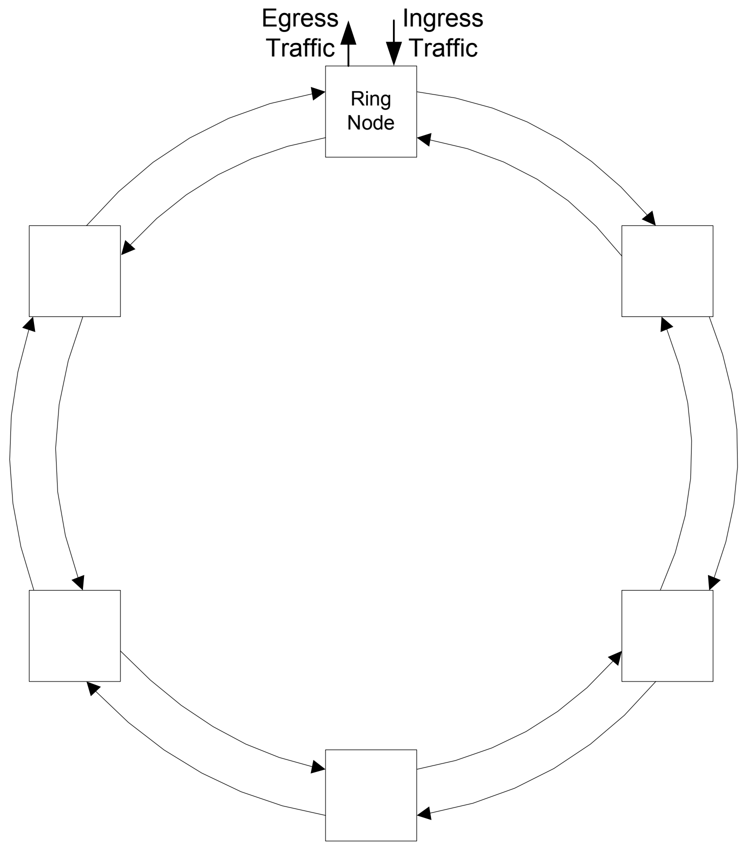
Problem Example

- MPLS VPN
  - Each VPN is a network user.
  - A VPN with more locations will have more flows, and receive a higher bandwidth allocation.



Problem Example

- Ring Network (RPR):
  - Fairness goal is to share bandwidth fairly among nodes (users)
  - Ingress-Egress flow fairness will reward nodes with multiple egress destinations.



Proposed User Fairness Criteria

- Goal:
  - Share fairly among users not flows
- Solution highlights:
  - Max-min type
  - No zero bandwidth allocation
  - Limit cases (one flow per each user, or only one user) converge back to original max-min.

Proposed User Fairness Criteria

- The work examines three fairness definitions that meet goals:
  - Weighted Max-Min.
  - Local (per-link) User Max-Min.
  - Redefined Vector Fairness.

Weighted Max-Min

- Use existent weighted max-min definition and algorithm.
- Define the weights inversely proportional to the number of flows per user:
  - $W_{ij}^{'u} = \frac{1}{n_u}$
  - The sum of weights per user is equal 1.
- No consideration of user bandwidth.

NOTE: Proposed by A. Banchs in “User Fair Queuing: Fair Allocation of Bandwidth for Users”.

Local (per-link) User Max-Min

- LUMM definition:
  - A flow can be increased unless it decreases a flow of another user which has a smaller user bandwidth on **a link** along the path.
- Defines user bandwidth allocation separately on each link.
- Non-finite algorithm (converging only).
- Uniqueness and existence are open questions.

NOTE: Presented as RIAS in "Design, Analysis, and Implementation of DVSR: A Fair, High Performance Protocol for Packet Rings" by Gambiroza, Yuan, Balzano, Liu, Sheafor, Knightly

Vector-space Model

- Max-min fair bandwidth allocation is equivalent to maximizing the lex-max lex-ordered vector in the space:

$$X = \{\vec{x} : (x_1, \ldots, x_N)\} \qquad x_n = f_{ij}^u \ \forall i, j, u \ 1 \leq n \leq N$$

f_{ij}^u	Bandwidth allocation to individual flow
N	total number of flows

Vector-space Model

- What if we extend the vector space to include the total user bandwidth?
- The lex-max lex-ordered vector is regular max-min fair.

$$X = \{\vec{x} : (x_1, \ldots, x_N, x_{N+1}, \ldots, x_{N+U})\} \qquad x_n = f_{ij}^u \ \forall i, j, u \ 1 \le n \le N$$

$$x_n = f^u \ \forall u \ N+1 \le n \le N+U$$

f_{ij}^u	Bandwidth allocation to individual flow
f^u	Bandwidth allocation to user
N	total number of flows
U	total number of users

Vector-space Model

- Add normalization factors:

$$X = \{\vec{x} : (x_1, \ldots, x_N, x_{N+1}, \ldots, x_{N+U})\} \qquad x_n = \alpha^u f_{ij}^u \;\; \forall i, j, u \;\; 1 \le n \le N$$

$$x_n = \alpha^u \beta^u \gamma f^u \;\; \forall u \;\; N+1 \le n \le N+U$$

α^u normalize between individual allocations of different users

β^u normalize between individual and user allocation

γ Factor relative importance of user vs. individual allocation

- The lex-max lex-ordered vector in any such space will match the fair bandwidth allocation for a corresponding fairness definition.

Redefined Vector-space Fairness

- Choosing the following weights:

$$\alpha^u = n_u \qquad \beta^u = 1/n_u \qquad \gamma = 1$$

- Results in a definition which maintains similar relations between individual flows like WMM, while extending the criterion to include dependence on the actual bandwidth allocated to the user.

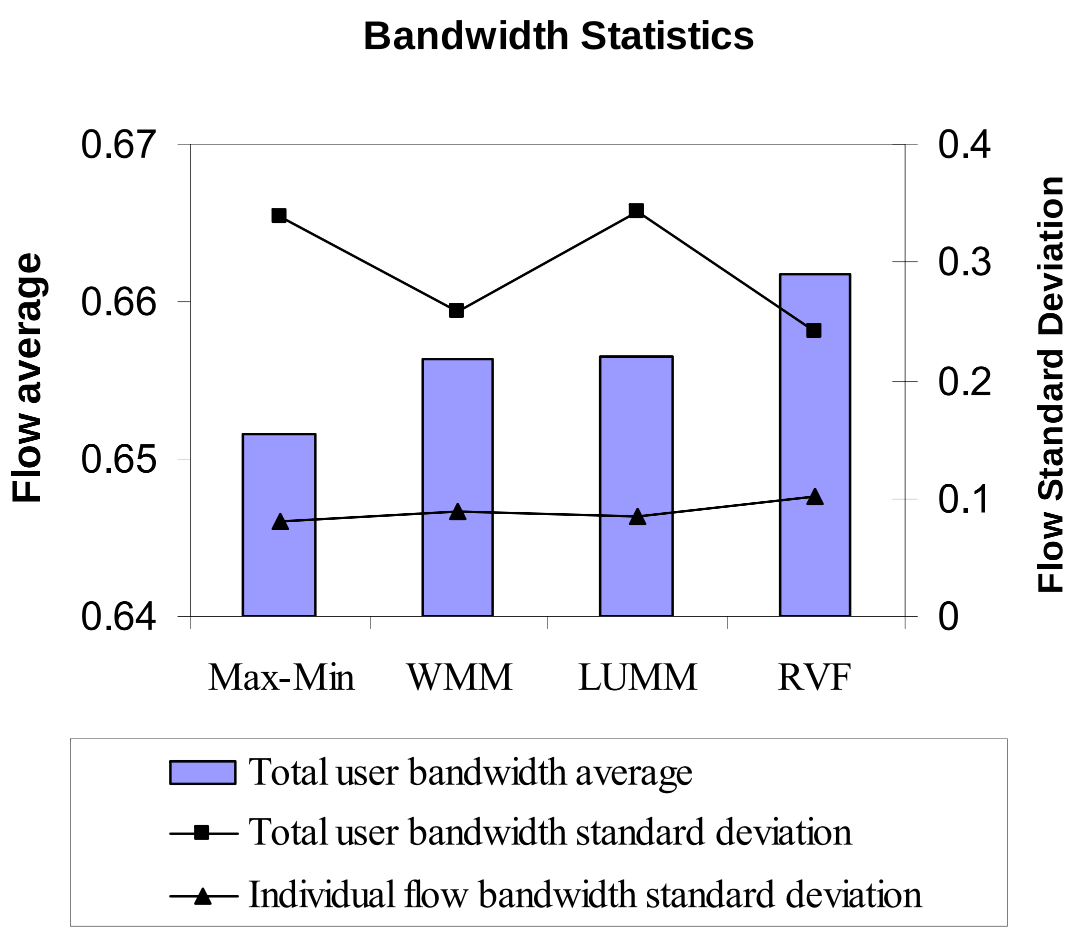
Redefined Vector-space Fairness

- Using results from "A Unified Framework for Max-Min and Min-Max Fairness with Applications" by La-Boudec et. al. we:
  - Prove uniqueness and existence of the fair allocation
  - Construct the algorithm to find the fair allocation.

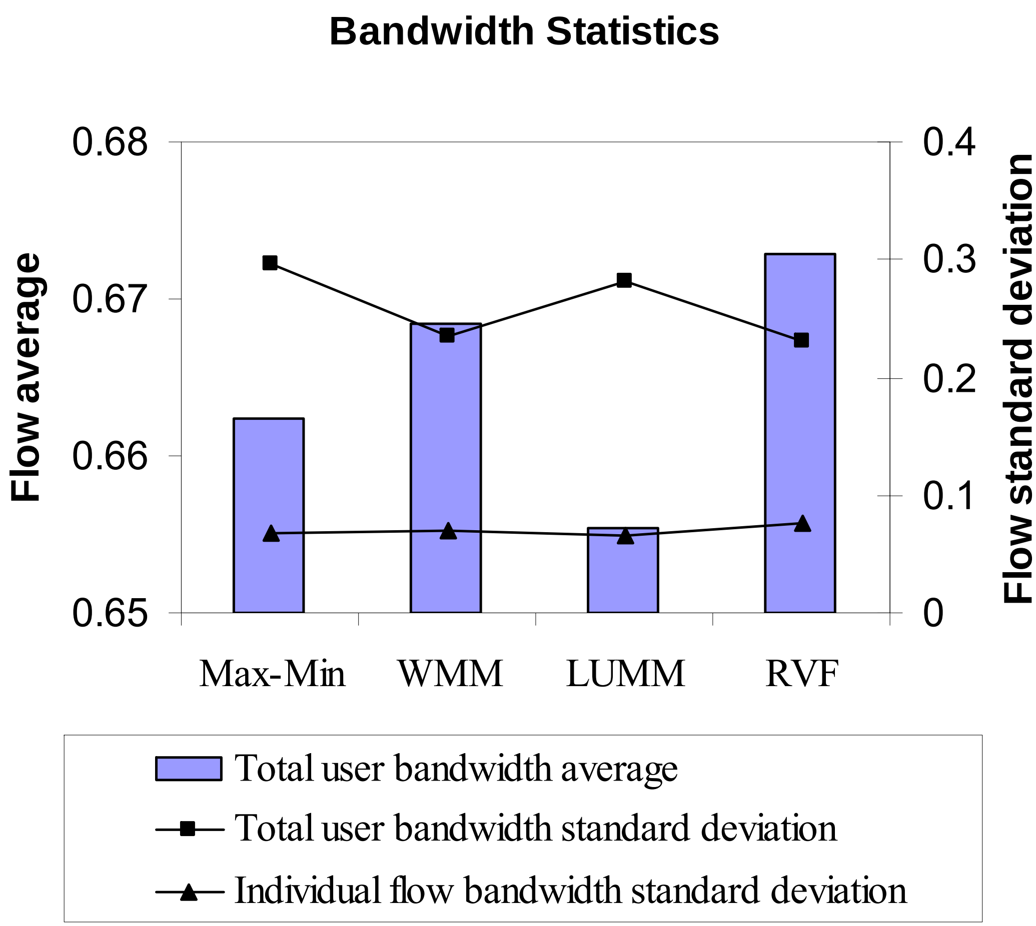
Performance Evaluation

- Measure "user fairness" by the standard deviation of the user bandwidth:
  - Identical networks.
  - Fixed number of users.
- Examples include:
  - Mesh or Ring network.
  - Multiple vs. single source per user.

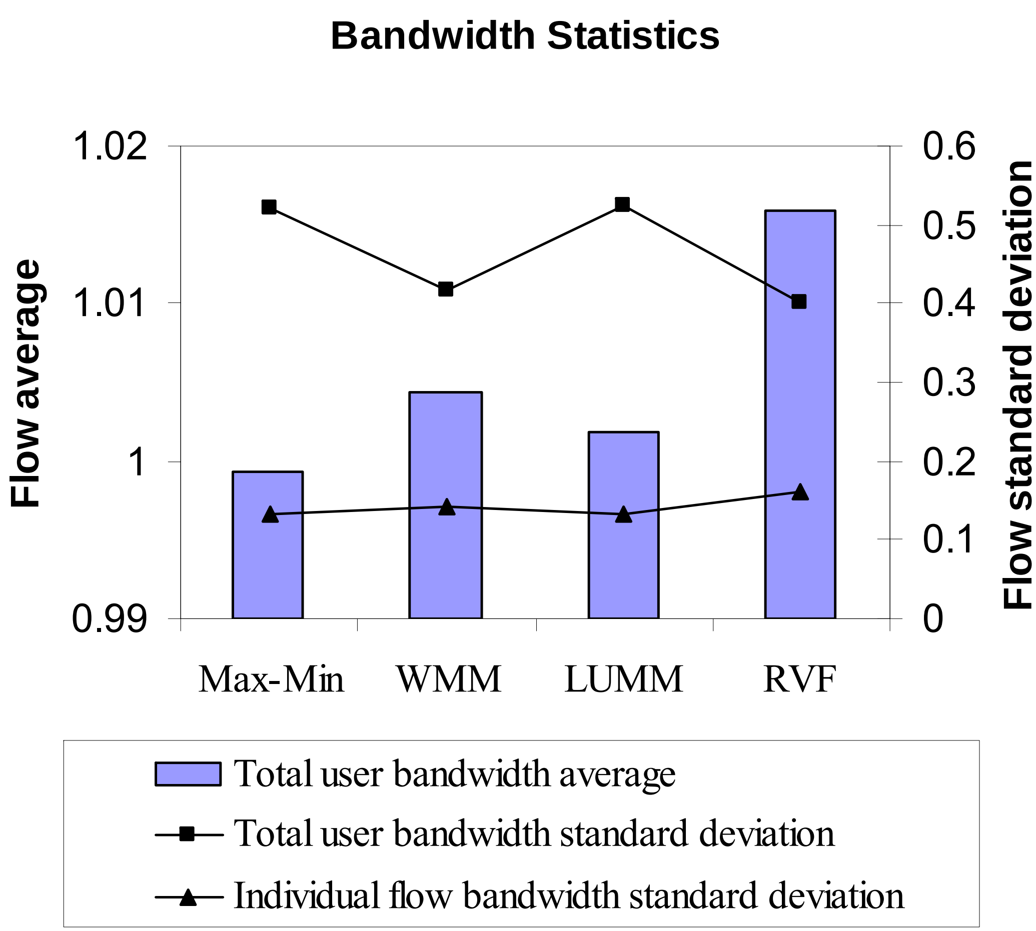
Mesh network, multiple sources per user



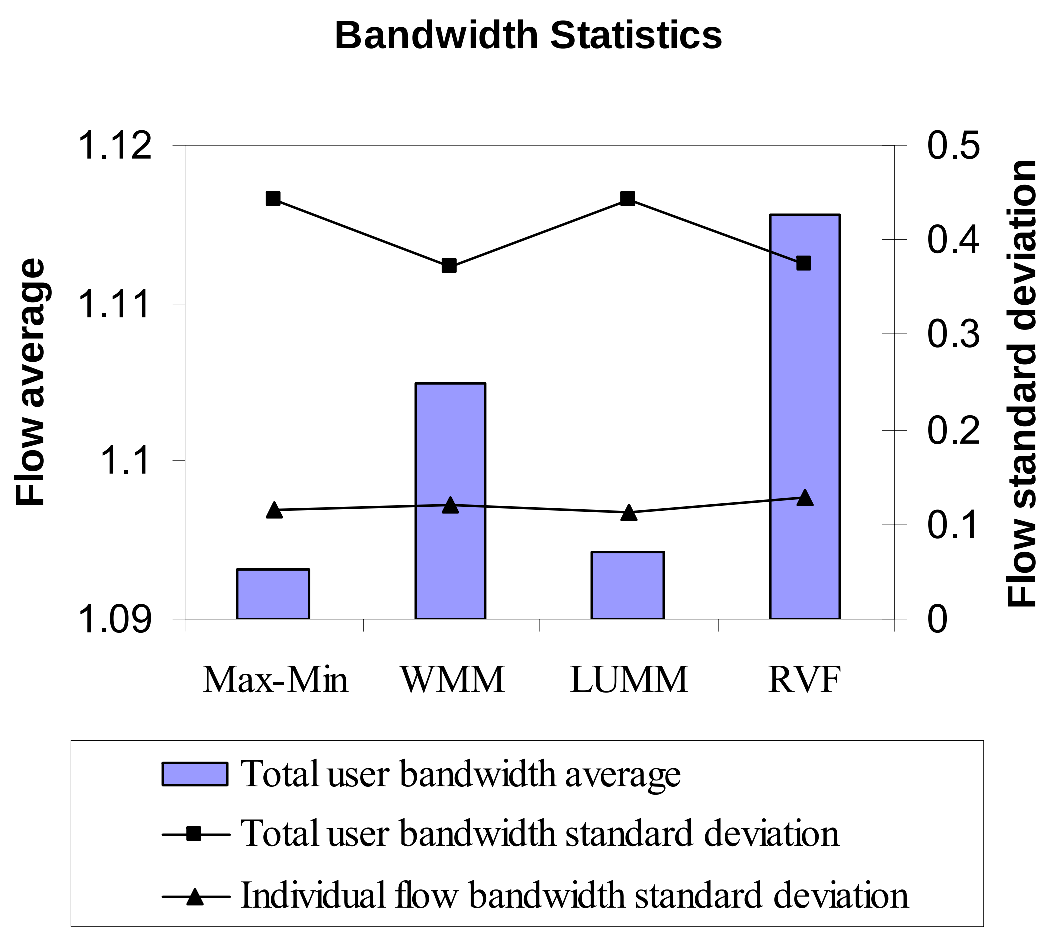
Mesh network, single source per user



Ring network, multiple sources per user



Ring network, single source per user



Conclusions

- We presented the new RVF user fairness definition:
  - Takes into consideration the total bandwidth allocated to the user.
  - Has an efficient algorithm for finding the fair allocation.
- We compared between RVF, WMM and LUMM user fairness criteria:
  - RVF gives the best average user fairness.
  - RVF results in the highest average network throughput.

The End

- Question and comments are welcomed:

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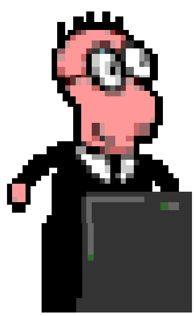

Math. Background

- Lexicographical ordering
  - {1, 3, 2.4, 1.7} → lex-order → {1, 1.7, 2.4, 3}
- Lexicographical comparison
  - $\{1, 2, 2.1, 3\} \overset{\text{lex}}{<} \{1, 2.1, 2.2, 2.5\}$

LUMM Algorithm

- Iterative algorithm
- Each step calculates the user fair rate per link.
- For each user the individual flows allocation is calculated using regular max-min algorithm, by setting the fair rate as the link capacity.
- Repeat until bandwidth allocation stops changing.

LUMM Algorithm

1. Initialization
   a. For each link l calculate User Fair Rate
      $FR_l = \frac{C_l}{N_l}$, N_l is the number of users on link l
   b. For each user calculate individual flow allocation by max-min algorithm, while setting each link capacity to FR_l - $\text{maxmin}\{f_{ij}^u, FR_l\}$
2. For each link l
   a. Sort all user flow on the link ascending, such that $f^1 \le f^2 \le \ldots \le f^{n_l}$
   b. if $\sum_{F_l} f_{ij}^u < C_l$ then $FR_l = \frac{C_l - \sum_{F_l} f_{ij}^u}{n_l}$, $n_l \ \ge \ |E|+1 \quad E = \{f^i : f^i = f^{n_l}\}$

      else:

      i. $N = N_l,\ C = C_l,\ i = 1,\ FR_l = C_l / N_l$

      ii. While $f^i < FR_l$ and $f^n \ge FR_l$
         1. $C = C - f^i$
         2. N=N-1
         3. $F_l = C / N$
3. For each user calculate individual flows - $\text{maxmin}\{f_{ij}^u, FR_l\}$ (same as step 1.b.)
4. If each flow f_{ij}^u has a bottleneck link end
   else repeat from step 2.

RVF Algorithm

$$X = \{x_1, \dots, x_n\} = \{\frac{f_{ij}^u}{W_{ij}^u}, \dots, \frac{f_{mn}^v}{W_{mn}^v}, \frac{f^u}{W^u}, \dots, \frac{f^v}{W^v}\}$$

$$I = \{1, \dots, N+U\}$$

f_{ij}^u is an individual flow

f^u is total user flow

N – number of flows

U – number of users

X is a subset of $\mathbb{R}^{N+U}$ with the limitation:

for each link l $\sum_{x_i = f_{ij}^u / W_{ij}^u \in F_l} W_{ij}^u x_i \le C_l$

for each user u $\sum_{x_i = f_{ij}^u / W_{ij}^u \in F^u} W_{ij}^u x_i = W^u x_j$

1. $I^0 = I; X^0 = X; n = 0$
2. while $I^n \ne \phi$ do
3. $n = n + 1$
4. Solve linear programming problem.

   maximize T^n subject to:

   $T^n \le x_i, \forall i \in I^{n-1}$

   $\vec{x} \in X^{n-1}$
5. In case solution is not a single value of $\vec{x}$, choose $\vec{x}$ that maintains fairness
6. $X^n = \{\vec{x} \in X^{n-1} \mid x_i \ge T^n \ \forall i \in I^{n-1}, \ \exists i \in I^{n-1} \ x_i > T^n\}$

   $I^n = \{i \in \{1, \dots, N\} \mid (\forall \vec{x} \in X^n)\, x_i > T^n\}$
7. Return the only element in X^n

RVF Algorithm

1. For each $x_i \in X^n$ that can't be increased

   2. If $x_i = f^u$ (user flow),

      $$x_k = \frac{x_i}{n_i} \cdot n_u \ ; x_k \ \square \ f_{ij}^u \cdot n_u \quad \forall f_{ij}^u \in F^u \ \& \ x_j \in X^n \text{(individual flow)}$$

      n_i is number of individual flows of user u still in X (not bottlenecked)

   3. If $x_i = n_u \cdot f_{ij}^u$ (individual flow of user u)

      a) $x_j = x_i$, for each individual flow $x_i \in X^n$, and belonging to same user u

      b) calculate user flow x_k based on the above allocation for individual flows